

# Math Institute Challenge

## Part I Record a video

In the video, first introduce yourself, including your name, grade level, and which Avenues campus you are applying from. Then present your responses to the following:

1. Create six circular disks that satisfy the following conditions:
  - a. Five of them are congruent to each other and the sixth one is larger than the others.
  - b. When the five circular disks are placed on the sixth, there is no space left uncovered. For example, if the large disk is painted in red, after the covering process no red should be visible at all.
  - c. When the large circle is completely covered, the five congruent disks are symmetrically placed so that they all intersect at the center of the large circle.
  - d. For any given large circle with radius  $R$ , the five congruent circles must have a minimum radius,  $r$ , for which the complete covering of the large circle is possible.
2. What is the maximum ratio of  $R$  to  $r$ ?
3. What is your process in finding the lengths of radii and creating the discs? Give detailed mathematical explanations along with any sketches or drawing you have.
4. If we remove the condition that the disks must be placed symmetrically, i.e., the five congruent disks intersect at the center of the large circle, how does that change the ratio? If you can come up with a mathematical solution and find the new ratio, that is great. If you cannot do so, you can still support your argument with a carefully drawn figure where the steps to draw it are clearly listed in your work.

## Part II Be prepared to discuss the following questions

1. Can a line segment be divided such that the ratio of the larger segment to the smaller is equal to the ratio of the whole line to the larger segment? If so, provide work to support your answer.
2. Consider the Fibonacci sequence. What is the limit of the ratio between each successive term, larger to smaller?
3. How do these two problems connect to the circle problem?

